Year 4 Class 102 (2008-2009 school year)

Convexity. Part 3.

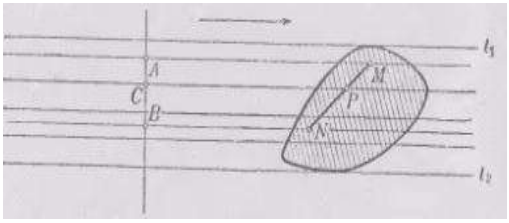
Def. A figure A in Euclidean space is ***bounded*** if it completely contained inside some circle.

Def. A line is called a ***supporting line*** to figure A if it passes through at least one boundary point of a figure, but the entire figure is located on one side of the line.

It is kind of obvious that any convex figure has no more than two parallel supporting lines. Then the figure us kind of sandwiched between these two lines.
Could you think of an example of a figure that has just one supporting line? No supporting lines?

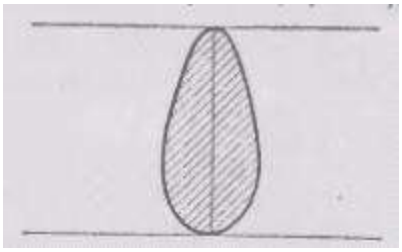
Theorem: Any bounded convex figure A with a nonempty interior has *exactly* two parallel supporting lines that are parallel to a given line.

Proof:



Theorem: For a convex bounded figure, consider the pair of supporting lines that are on maximal distance one from another. Then each of these two lines contains a single point of the boundary, and a segment connecting these two points is perpendicular to these supporting lines

Proof:



Theorem: For a convex bounded figure, max distance between any two points of this figure coincides with the maximal distance between its supporting lines.

Proof:

Def. A ***diameter*** of a figure is a maximal distance between points of this figure.