

Year 4 Problem Set 105 (2008-2009 school year).

End of the Year Olympiad.

1. A forest consists of furs and birches. Company “Ross-Stump” had cut one third of all furs and one sixth of all birches. The ecological organization “Green Warrior” is claiming that half of all the trees in the forest had been cut. Is “Green Warrior” report correct?
2. Sum of three natural numbers equals 100. Three pair-wise differences of these numbers were taken (a smaller one was always subtracted from a bigger one). These three differences were added. What is the maximal possible sum of these three differences?
3. A peg is placed on the leftmost square of a 1 by 40 board. Sasha and Masha are playing a game. On a turn, a player can move the peg either to the left or to the right. The only condition is that a length of a move can't be used up more than once. A player who is not able to make a move loses the game. Who has a strategy to win: Sasha or Masha?
4. Hooligan Vasia switched the tags on button in the elevator of a 13-story building. Now the floor that the elevator is going to not always coincides with the tag on the button that had been pressed. Little Bella had spend the whole day playing in the elevator. She played it like this: if the elevator stopped at the N s floor, she always pressed the button with the tag “ N ”. She played this whole day. By the end of the day Bella has found out that whatever floor she starts at she always gets back to the same floor after she presses the button 1313 times. Prove that playing like this Bella is always able to get from any floor at the building to any other floor.
5. Prove that any natural number can be expressed as a quotient of two integer numbers: fifth exponent of some natural number and cube of some natural number.
6. A chess board has size 100 by 100. This chess board was cut into certain number of squares with sides of odd length. A central point was marked in each square. Prove that total amount of marked black squares equals to total number of marked white squares.

