

Year 4. Problem Set 91 (2008-2009 school year).

1. Farmer John is inviting his grandson Jake to visit his farm. He told the boy that he had planted a new fruit garden of apple and pear trees. He also told that in his garden every apple tree had exactly two pear trees planted 10 meters from it. Jake was not impressed. "Your garden has two times as many pear trees as apple trees", he said. Farmer John smiled and replied that, on the contrary, there were twice as many apple trees as pear trees. How could that be possible?
2. Factor $x^{12} - y^{12}$ as completely as possible with integral coefficients and integral exponents.
3. If the sum of two numbers is 1 and their product is 1, then what is the sum of their cubes?
4. A triangle ABC is inscribed into a circle. A bisector of angle B intersects this circle in the point M . Point O is the center of the circle that is inscribed into triangle ABC . Prove that points A, C and O are all located on a circle with center M .
5. Each point on a straight line is painted in one of two colors. Prove that there exist three same-color points A, B and C on this line such that B is the center of the segment AC .
6. Sasha and Tim have two white quad-lined paper squares of size 8×8 . Each of them has an equal number of unit squares of their rectangles colored in blue. The boys would like to cut their squares into 1×2 dominoes and then rearrange the dominoes into 8×8 squares in such a way that they both would have the same blue picture. Prove that they can always do that.

