Year 4 Problem Set 92 (2008-2009 school year)

- 1. Cut the circle into several equal parts in such a way that there is at least one part that doesn't contain the circle's center.
- 2. You are learning to play a game of battleship. Your opponent has placed a single battleship (sized 1x4) on a 10x10 grid. What is the smallest number of shots you should fire to guarantee that you definitely hit it?



- 3. A circle is inscribed into a triangle ABC. It touches the sides of the original triangle at points M, N, P. Prove that all angles of the triangle MNP are acute.
- 4. Prove that for any *n* the *next to last* digit of 3^n is always even.
- 5. In this problem, "*word*" means any sequence of letters. Suppose that we have two words u and v such that uv = vu. Prove that there exists such a word s that both words u and v actually consist of a word s repeated a few times.
- 6. During a parliamentary debate on the planet Pluke, all members engaged in a quarrel. While quarrelling, every member spat onto exactly one other member. Prove that now all the members of parliament can separate into three fractions in such a way that within each fraction nobody has spitted at any other member of the same fraction.

