## Year 4 Problem Set 95 (2008-2009 school year)

1. Can you paint 4 vertices of a cube red and 4 vertices blue in such a way that any plane passing through any three vertices of a same color contains a vertex of different color?

2. An island of Polygonia is shaped as a polygon (not necessary convex). This island is divided into a few countries. They all have triangular shapes, and any two neighboring countries have a whole common side. It means that no country triangle have on its side a vertex of another triangle. Prove that you can paint the map of this island into three colors in such a way that every country is assigned its own color, and no two neighboring countries are painted in the same color.


3. Prove that $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3$ for any $a,b,c\geq 0$.

4. Prove that $3x^3-6x^2+4\geq 0$ for any $x\geq 0$.

5. A hexagon $ABCDEF$ is inscribed in a circle. Diagonals $AD, BE$ and $CF$ are diameters of this circle. Prove that the area of the hexagon $ABCDEF$ is twice the area of the triangle $ACE$.

6. $ABCD$ is a square. Points $a,b,c,d$ are marked on sides $AB,BC,CD,DA$ correspondingly in such a way that $|Aa|=|Bb|=|Cc|=|Dd|$ . What should $|Aa|$ be equal to if we want an area of the figure $abcd$ to be as small as possible?