Year 4 Problem Set 98 (2008-2009 school year)

- 1. Can you find 4 numbers such that their 6 pair-wise differences (in fact, absolute values of differences) are 2,2,3,4,5,6?
- 2. King Charlemagne has 2500 bottles of wine in his cellar. He drinks wine that came from Spain, France and Italy. Spaniards bottle their wine in 0.5 liter bottles, Italians in 0.7 liter bottles, and French in 1 liter bottles. The king has 2006 liters of wine total. Prove that he has at least one bottle of Spanish wine.



3. At the court of the prince Lemon there were three types of courtiers: barons, counts and dukes. At the very beginning of the prince's reign there were 2006 of them total, but every day one of the courtiers was



killing another one on a duel. It happened that dukes were killing only counts, counts were killing only barons, and barons were killing only dukes. Nobody has ever won a duel twice. At the end, the only survivor was Baron Orange. Who was killed at the very first duel: baron, duke or count?



- 4. Sasha wrote number 2 on a whiteboard. In fact, he did it 10 times. Then he told Masha that she can erase any two numbers and write their sum or product instead. She can repeat this operation a few times. Could she end up with a single number 1002 on the whiteboard?
- 5. Prove that any rectangle can be cut into a few parts in such a way that a rectangle with the side of length 1 can be constructed from these parts.