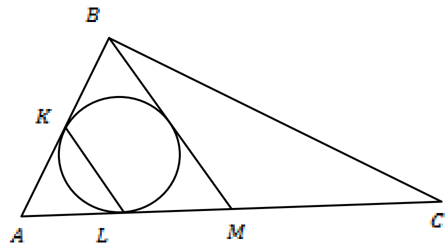
Year 5. Problem Set 106 (2009-2010 school year).

Start of the Year Olympiad.

1. Before the battle, Robin Hood and Little John had equal number of arrows in their quivers. During the battle, Robin Hood used up 8 times less arrows then Little John. As a result, Little John was left with 9 times less arrows then Robin Hood. Prove that they originally had at least 70 arrows each.

2. Three missionaries and three cannibals are going to cross the river. They have a single boat that fits not more than 2 people. Missionaries are scared of being outnumbered by cannibals, so at no point in time can there fewer missionaries then cannibals on any side of the river. Only one missionary and one cannibal know how to row. Show how they can cross the river. Note: assume that every time a boat arrives to a shore, everybody leaves the boat. The conditions "no fewer missionaries then cannibals on the shore" should hold at this moment as well.

3. In the triangle ABC angle B is 90 degrees. BM is a median. K and L are the points where the circle inscribed into triangle ABM touches sides AB and AM. It is known that KL is parallel to BM. Find the angle ACB.



4. Five-digit number A is written using only digits 2 and 3. Five-digit number B is written using only digits 3 and 4. Could it be that the product of these numbers has digit 2 only in its decimal notation?

5. On a white 9 by 9 board, 19 one by one squares have been colored red. Prove that either there exist 2 small red squares that share a common side, or there exist a white square with at least two red neighbors (neighboring squares are supposed to share a side).

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6. A natural number x was divided by $3, 18$ and 48. The remainders of all three divisions were added. It happened that this sum was equal to 39. Prove that $x - 1$ is divisible by 3.

7. Function $f(x)$ is defined for any real number x. For any real x the following inequalities are true:

$$f(x+1) \leq f(2x+1), \qquad f(2x+1) \geq f(4x+1)$$

It is also known that $f(3) = 2$. Prove that the equation $f(x) = 2$ has more than a million solutions.