Year 5. Problem Set 109 (2009-2010 school year).

- 1. If A and B are two sets, and $A \subset B$, then what is AB? Prove the answer using the picture and using "x belongs to a set" notation.
- 2. Using "x belongs to a set" notation prove that for any sets *A*, *B*, *C* and universal set *I*
 - a) $A + \overline{A} = I$
 - b) $\overline{\overline{A}} = A$
 - c) A(B+C) = AB + AC
- 3. Prove the following equalities and simplify expressions (you can use pictures as a proof)
 - a) $\overline{A + B} = \overline{A}\overline{B}$ b) $\overline{AB} = \overline{A} + \overline{B}$
 - c) $(A + B)^n =$
- 4. On the island of Puhu-Puhu, 2/3 of all men are married, and 3/5 of all women are married. What fraction of the island population is married?



- 5. Anton, Tim and Alex together solved the set of 100 problems. Each of them solved 60 problems. A problem is called "hard" if it was solved by one student only. A problem is called "easy" if it was solved by all three. Find the difference in the amount of hard and easy problems in the set.
- 6. The Intergalactic Community College offers 300 classes. This year, all classes are full, and each has 15 students registered. Some of the students are from Alpha Centauri; they have several hundred brains and usually attend all classes at once. No other race in the galaxy can do that.

The professor of Mathematics noticed that if you take any two classes, there are exactly 29 students registered for either of them. How many Alpha Centaurians have registered this year?

