

Year 5. Problem Set 110 (2009-2010 school year).

1. It is known for the sets X , Y and A that $X \cup Y = A$ and $X \cap A = Y$. Find the sets X and Y .
2. Prove that the set of points on the plane that have two integer coordinates is countable.

3. The knights of King Arthurs' court formed a lot of different alliances, mostly created in order to serve this or that lady. It is known that for any two alliances A and B the set of knights that is formed as $\overline{A \cup B}$ is an alliance as well. This fact is true even if A and B are the same alliance. Prove that for any two alliances A and B the set of knights $A \cup B$ is an alliance as well.



4. A set C contains n elements. In how many ways can you choose two subsets A and B of the set C in such a way that:
 - a) The intersection of A and B is empty
 - b) The set A contained in the set B
5. 2005 numbers are placed in a row. The first number equals 2006. It is known that for every number except for the first one and the last one, the number is equal to the sum of its neighbors. What is the last number in the row?

6. The parliament's elections are taking place on the island of Puhu-Puhu. Every ballot contains the same list of n candidates. Every voter marks in his ballot the names of some of these n candidates. Each voter marks at least one candidate. The island has a single polling station with $n + 1$ election booths. Each



booth has a box for ballot collection. After the elections were over, it turned out that every box has at least one ballot. Also it turned out that whichever way you take one ballot from every box the resulting set of $n + 1$ ballots always has at least one commonly marked candidate. Prove that there exists at least one box with the same candidate marked in all ballots in this box.