Year 5. Session 110 (2009-2010 school year).
Set Theory.

How to Compare Sets? Equal sets.

Finite sets are of equal size if it is possible to find one-to-one correspondence between elements of these sets.

Infinite sets: when are they equal?

Notation:
Def: Two infinite sets A and B are ***equal*** if there it is possible to find one-to-one correspondence between the elements of these sets.
Terminology: Sets A and B have ***same cardinality***, Sets A and B have ***are equivalent.***

Dogma: a part is less that the whole. True?

Fact: Any infinite subset of the set of natural numbers is equivalent to the set of natural numbers.

Def: Any set that is equivalent to the set of natural numbers $\mathbb{N}$ is called ***countable***.

Problem: Prove that a set if integer numbers $\mathbb{Z}$ is equivalent to the set of natural numbers $\mathbb{N}$.

Theorem: The set of all rational numbers is countable. (The set of natural numbers is equivalent to the set of rational numbers).
Proof:

Theorem: A sum of any number of countable sets is countable.
Proof:

Non equal sets

Def: A set *A* has at least as many elements as set B if *B* is equivalent to a subset of A (cardinality of B is not greater than cardinality of A)

Lemma: Any infinite set has a countable subset.
Proof:

Theorem: Countable set is a smallest of all the infinite sets. (A cardinality of a countable set is not greater that the cardinality of any other infinite set).

Theorem: Cardinality of an infinite set doesn't change if we add a countable set to it.

Question: Are all infinite sets countable?