## Year 5. Problem Set 112 (2009-2010 school year).

1. Alex plays with his wooden cubes. He constructs a parallelepiped out of these cubes. Alex took a piece of paper and wrote down the volume, surface area and sum of edge lengths of the parallelepiped. The numbers on the paper are: 42, 48 and 82 , but it is not known which number corresponds to which measurement. What are the dimensions of the parallelepiped?
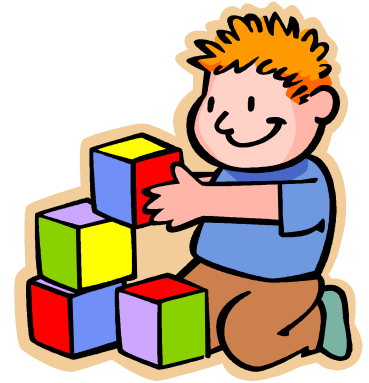
2. Prove that the cardinality of an infinite uncountable set does not change if we subtract a countable set from it.

   Comments: you should prove that if an infinite set A was equivalent to an infinite set B, and if a countable set C was removed from the set A, then the remaining set $A - C$ is still equivalent to set B.

3. Prove that the set of all points on the line is not countable (you cannot just use the fact that the set $(0,1)$ is not countable. You should prove it).

4. Consider two sets: a set of points on the interval $(0,1)$ and a set of points on the line $(-\infty, \infty)$. Prove that these sets have the same cardinality

5. Using a compass and a non-marked ruler, construct a triangle ABC from:

   - angle $\angle B$,
   - side ratio $|AB| : |BC| = 2 : 3$, and
   - the length of the bisector $|BD|$.

6. Prove that:

   a) All bisectors of a triangle intersect in a single point.
   b) Let $|BD|$ be a bisector of the angle $\angle B$ in a triangle ABC . Prove that $|AB| : |BC| = |AD| : |DC|$. (Hint: look at the triangles ABD and BCD).