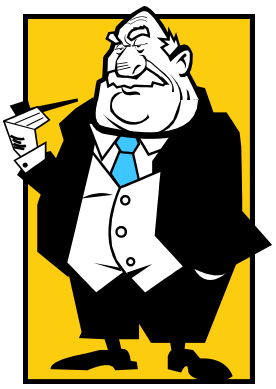
Year 5. Session 116. Olympiad. (2009-2010 school year).

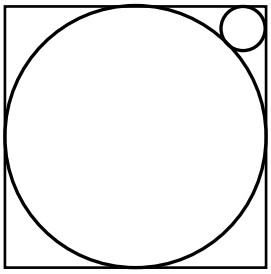
1. The ancient hero Hercules had fifty children – all sons. From all his descendants, some had never married and had no children and all. Those who married always had three children – all sons. If exactly 150 of Hercules's descendants had children, how many offspring did Hercules have?

2. There was a group of English gentlemen living on the island of Fiji in 1889. While some of these Englishmen knew one another, others did not. In order to facilitate friendly connections, these people started the tradition of parties. Each evening, one gentlemen would invite all his acquaintances for a party and introduce them all one to another. After a while, everybody has already hosted a party, but Mr. Pepperidge still was not introduced to Mr. Coolidge. Prove that if somebody decides to host another party, these two gentlemen would not be introduced one to another on this party either.



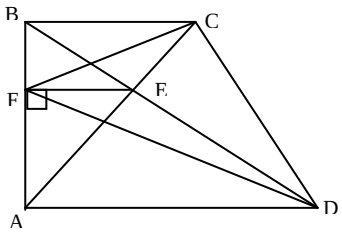
3. Somebody drew 9 vertical lines and 9 horizontal lines inside a square. As a result, the square was partitioned into 100 rectangles. It happened that exactly 9 of these rectangles were squares. Prove that two of these squares are of equal size. (problems.ru – set theory problems)

4. Calculate the radius of the smaller circle. Assume that the square has sides of unit length.



5. Somebody placed the numbers $1, \ldots, 64$ in the cells of an $8 * 8$ table. The numbers are placed in such a way that in each row they are decreasing from left to right and in each column they are decreasing from top to bottom. Prove that the sum of 8 numbers on the diagonal a8-h1 is not less than 204.

6. Consider a trapesoid $ACBD$ with bases BC and AD and with the angle A equal 90^o. The point E is the point of intersection of the diagonals, the point F is the projection of E onto side AB. Prove that the the angles CFE and DFE are eqaul.



7. 175 Humpties cost more than 125 Dumpties, but less than 126 Dumpties. The prices for Humpties and Dumpties are expressed in whole numbers of dollars. Prove that
   a. \$80 are not sufficient to buy 3 Humpties and 1 Dumpty.
   b. \$100 are not sufficient to buy 3 Humpties and 1 Dumpty.