Year 5. Problem Set 118 (2009-2010 school year).

1. The cube-shaped planet "Cube" is located in the remote corner of the Galaxy of Knights and Liars. Each face of this cube is actually an independent kingdom. A king, who is either a knight or a liar, rules each kingdom. On the annual leadership conference, each ruler on this planet claimed that most of his neighboring kings are liars. How many kings at this planet are knights?

2. A round table has been placed in the corner of the room in such a way that it touches both corner walls. A line that connects two tangent points was drawn. Then a point was marked at this line. The distance from this point to one wall is 10 centimeters, and the distance to the other wall is 18 centimenters. Find the radius of the circle.

3. The triangle ABC is isosceles. Its base is of unit length, and the angles at the base are α degrees each. The bigger circle touches all three sides of the triangle. The smaller circle touches two sides and the bigger circle. Calculate the radius of the smaller circle.
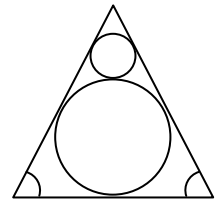
4. A 15-inch-long stick has four marks on it, dividing it into five segments of length 1,2,3, 4, and 5 inches (although not necessarily in that order) to make a "ruler." Here is an example: | 2 | 3 | 5 | 1 | 4 |. Using this ruler, you could measure 8 inches (between the marks *B* and *D*) and 11 inches (between the end of the ruler at *A* and the mark at *E*), but there is no way you could measure 12 inches. Prove that it is impossible to place the four marks on the stick such that the five segments have length 1,2,3, 4, and 5 inches, and such that every integer distance from 1 inch through 15 inches could be measured.

5. The points of the plane are colored in black and white so that whenever three vertices of a parallelogram are the same color, the fourth vertex is that color, too. Prove that all the points of the plane are the same color.

6. In the triangle *ABC*, *D* and *E* are two points on segment *BC* such that *BD* = *CE* and angle *BAD* is equal to the angle *CAE*. Prove that the triangle *ABC* is isosceles. BAmo 2007